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An Analysis of the Accelerated Expansion of the Universe in the Framework of Brans-Dicke Theory of Gravitation

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ABSTRACT: In the present study, we have accounted for the accelerated expansion of the universe in the framework of the generalized Brans-Dicke theory. It has been shown very clearly that the transition of the universe from a phase of deceleration to that of acceleration can be explained by the generalized Brans-Dicke theory, without having to consider any contribution from the entity called dark energy. The present model shows a signature flip of the deceleration parameter, from a positive to a negative value, occurring in the recent past of the matter dominated era. We have determined the functional form of the Brans-Dicke parameter, expressed in terms of the scalar field. This functional form causes the effective pressure to be sufficiently negative to drive the accelerated expansion of the universe. It has been found in the present study that the gravitational constant increases with time.

KEYWORDS: Cosmic Acceleration, Deceleration Parameter, Brans-Dicke Parameter, Scalar Field, Dark Energy, Gravitational Constant.

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1. INTODUCTION

It has been established beyond doubt, by recent astrophysical observations, that there was a smooth transition of the universe from a decelerated phase to an accelerated phase of expansion [1, 2]. The reason behind this accelerated expansion has been a matter of intense research for many years. It is known to us that normal matter interacts through the gravitational force and it has a positive definite density and pressure. Therefore, there must be some other kind of matter responsible for the accelerated expansion, which makes the effective pressure sufficiently negative, causing a repulsive effect. This new entity is generally referred to as dark energy. In the past few years a lot of research has been undertaken to analyse the nature of this new form of energy, responsible for accelerated expansion. A number of models have been proposed so far to account for the acceleration of the universe. The true composition and behaviour of this new entity is yet to be determined.

The most obvious choice to represent dark energy has been the cosmological constant, denoted by the symbol Λ (lambda) [3]. But there are serious problems regarding the value of this constant which can't be accurately determined by the ΛCDM model. The value of Λ, obtained from recent observations for an accelerating universe, is not consistent with the value in Planck scale or Electro-weak scale [4]. By constructing a model, where Λ is a function of time, this problem can be partly solved. Several such models have been formulated but they have their own shortcomings [5, 6].

Scalar field models have been proposed as an alternative to the dynamical Λ models. Here, the equation of state of dark energy is a function of time. Quintessence models were proposed as one of the most important types among the scalar field models. These models have a potential function $V(\varphi)$, enabling the pressure sector, which is a function of the scalar field, to have a sufficiently large negative value, leading to the predictions consistent with observations regarding the accelerated expansion of the universe [7, 8]. A major limitation of these quintessence models is that most of the quintessence potentials are arbitrarily chosen, without having any theoretical basis behind their formulation. Quite naturally, a large number of models, based on scalar fields, such as the tachyon, k-essence, holographic dark energy models have come into existence with their strengths and limitations [9, 10, 11].

The cold dark matter and dark energy are normally allowed to evolve independently in most of the scalar field models. There are some studies where one of them decays into the other through some interaction between them [12]. A non-minimal coupling of the scalar field with the dark matter sector, through an interference term in the action, has helped us to explain the accelerated expansion of the universe. These are known as chameleon fields and they can effectively serve as dark energy, causing the cosmic acceleration [13, 14].

In the framework of Brans-Dicke (BD) theory, non-minimal coupling between the scalar field and geometry, play a very important role in explaining the observed acceleration. A self-interaction potential $V(\varphi)$, which is a function of the BD scalar field φ, has often been incorporated in the framework of Brans-Dicke theory to explain the cosmic acceleration [15]. In some cosmological models, a quintessence scalar field is introduced in

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the BD theory. It can account for the late time acceleration for a wide range of potentials [16]. Assuming an interaction between the BD scalar field and dark matter, one can show that the matter dominated era can undergo a transition from a decelerated phase to an accelerated phase of expansion without using any additional potential [17]. It has also been found that the BD scalar field alone can cause the acceleration without having to consider any quintessence matter or any interaction between BD field and dark matter [18].

In contrary to the astrophysical observations, many of these models predict an ever accelerating expansion for the matter dominated universe. To account for the recent acceleration, many of the models require a very low value of the Brans-Dicke parameter ω, of the order of unity, but the local astronomical experiments require a very high value of ω [19].

In a recent study on $f(R,T)$ gravity, it has been shown, that the time evolution of cosmological parameters can be correctly predicted without having the necessity to incorporate any cosmological constant or quintessence field in the model [20]. The entire cosmological scenario has been derived from the relation $f(R,T) = R + \alpha T + \beta T^2$. Here R and T denote the Ricci scalar and trace of the energy-momentum tensor respectively. This model shows a smooth transition of the universe from a phase of decelerated expansion to the phase of accelerated expansion, indicated by a change of sign of the deceleration parameter (q). Another recent model, based on BD theory, has shown this signature flip of q, by assuming an inter-conversion between matter and dark energy [21]. These models have been constructed for a matter dominated universe.

The purpose of the present work is to study the acceleration of the universe in the framework of the Generalised Brans-Dicke theory. This version of the BD theory was first proposed by Bergmann and Wagoner [22, 23]. In the generalised form, the BD parameter ω is regarded as a function of the BD scalar field (φ), instead of being a constant. For various physical requirements, different functional forms of ω would originate. Here we have tried to find out whether a time-varying ω can overcome the problems of usual BD models and lead to a smooth transition from the phase of decelerated expansion to accelerated expansion of the universe.

In Section 2 of this article, we have written the field equations of the generalized BD theory and derived the scale factor (a). We have used an ansatz for the scalar field (φ), expressed in terms of the scale factor (a). In Section 3, we have analysed the deceleration parameter (q). An expression for $\omega(\varphi)$ has been derived in section 4. In Section 5, we have discussed the time evolution of the gravitational constant. Section 6 has some discussions regarding the results obtained from this model.

2. BRANS-DICKE FIELD EQUATIONS AND THE SCALE FACTOR

According to the generalized Brans-Dicke theory, the action is given by [24],

$$S=\int \left[\frac{\varphi R}{16\pi G} + \frac{\omega(\varphi)}{\varphi}\varphi_{,\mu}\varphi^{,\mu} + L_m\right]\sqrt{-g}\,d^4x \tag{1}$$

Here, R denotes the Ricci scalar, L_m denotes the matter Lagrangian, φ represents the Brans-Dicke scalar field and ω is a dimensionless coupling parameter which is regarded as a function of φ here, instead of being treated as a constant.

For a zero-curvature Robertson-Walker space-time, the line element is expressed as,

$$ds^2 = dt^2 - a^2(t) f(r,\theta,\phi) \tag{2}$$

where, $f = dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)$

Here, a is the scale factor of the universe. Variation of action (1) with respect to the metric tensor components, given by equation (2), yields the following field equations.

$$3\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{\varphi} + \frac{\omega(\varphi)}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - 3\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} \tag{3}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\frac{\omega(\varphi)}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - 2\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} - \frac{\ddot{\varphi}}{\varphi} \tag{4}$$

In equation (3), ρ denotes the density of the entire matter distribution and $8\pi G = 1$. An overhead dot stands for differentiation with respect to time. This model is based on an assumption that the thermodynamic pressure of the cosmic fluid is zero, which is quite consistent with the present dust universe. To make the above differential equations tractable, we have chosen the following relation.

$$\varphi = \varphi_0\left(\frac{a}{a_0}\right)^n = \varphi_0\left(\frac{a}{1}\right)^n = \varphi_0 a^n \tag{5}$$

Here a_0 is the scale factor at the present time. Its value has been chosen to be 1, as per convention. Considering the conservation of matter, we can write, $\rho = \frac{\rho_0 {a_0}^3}{a^3} = \frac{\rho_0}{a^3}$

Combining equations (3) and (4) and substituting from equation (5), one gets,

$$\frac{\ddot{a}}{a} + (n+2)\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{n+2}\frac{\rho_0}{\varphi_0}a^{-(n+3)} \tag{6}$$

The solution of equation (6) gives the following expression for the scale factor.

$$a = \left[\frac{(3+n)\{(t+C_2)^2{\rho_0}^2 - C_1{\varphi_0}^2(2+n)^2\}}{2(2+n)\varphi_0\rho_0}\right]^{\frac{1}{3+n}} \tag{7}$$

Here, C_1 and C_2 are the integration constants. To determine the values of these constants we have used the following boundary conditions.

1) at $t = t_0$, $a = a_0 = 1$ and 2) at $t = t_0$, $\dot{a}/a = H_0$

Here t_o denotes the present epoch. Using these boundary conditions, the constants C_1 and C_2 are found to have the following forms.

$$C_1 = H_0^2 - \frac{2\rho_0}{\varphi_0(2+n)(3+n)} \tag{8}$$

$$C_2 = \frac{H_0\varphi_0(2+n)}{\rho_0} - t_0 \tag{9}$$

Using equations (8) and (9) in (7) we get the following expression for the scale factor.

$$a = [a_1 + a_2]^{\frac{1}{3+n}} \tag{10}$$

where, $a_1 = 1 + H_0(3+n)(t - t_0)$

and, $a_2 = \frac{\rho_0(3+n)(t-t_0)^2}{2(2+n)\varphi_0}$

The variation of scale factor (a), as a function of $(t - t_0)$, where t_0 denotes the present epoch, has been shown in Figure 1, for two values of the parameter n.

3. TIME EVOLUTION OF THE DECELERATION PARAMETER (q)

Using equation (10), one obtains the following expression for the deceleration parameter.

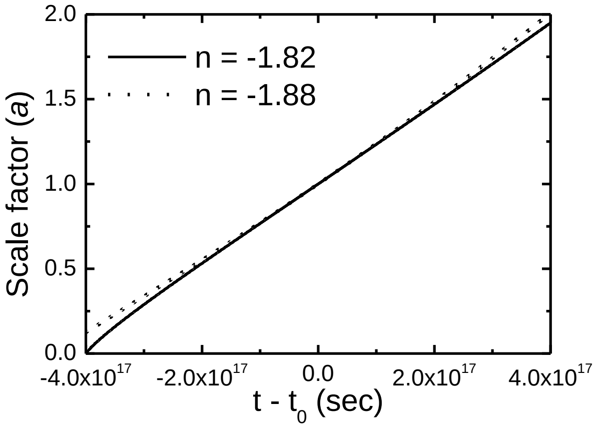

Figure 1: Variation of the scale factor (a) as a function of $(t - t_0)$, where t_0 denotes the present epoch, for two values of the parameter *n*.

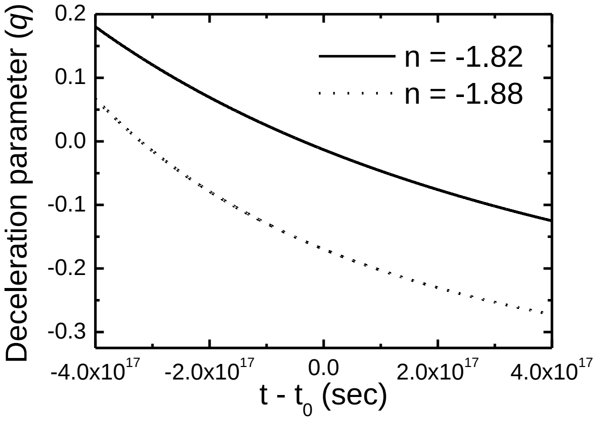

Figure 2: Variation of the deceleration parameter (q) as a function of $(t - t_0)$, where t_0 denotes the present epoch, for two values of the parameter *n*.

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}(q_1 + q_2) \tag{11}$$

Here, $q_1 = 1 + n$, $q_2 = \frac{(2+n)\varphi_0\{\varphi_0 {H_0}^2(3+n)(2+n) - 2\rho_0\}}{\{(2+n)\varphi_0 H_0 + \rho_0(t-t_0)\}^2}$

The transition of the universe, from a decelerated to an accelerated phase of expansion, indicates clearly that the deceleration parameter q was initially positive, which became zero at a certain value of t and has attained a negative value thereafter, in the recent past of the matter dominated era. This implies that the present value of the deceleration parameter (i.e. the value of q at $t = t_0$) should be negative. From equation (11), we have determined the time (T) at which q was zero. It is expressed as,

$$T = t_0 - f_1 + f_2 \tag{12}$$

where, $f_1 = \frac{H_0 \varphi_0}{\rho_0}(2+n)$

and, $f_2 = \sqrt{\frac{\varphi_0(2+n)[2\rho_0 - {H_0}^2 \varphi_0 (n+2)(n+3)]}{{\rho_0}^2(1+n)}}$

It is obvious from astrophysical observations that $T < t_0$ .This requirement is fulfilled if the parameter n satisfies the following condition.

$$n_- < n < n_+ \tag{13}$$

where, $n_- = -2 - \sqrt{\frac{\rho_0}{\varphi_0 H_0^2}}$

and, $n_+ = -2 + \sqrt{\frac{\rho_0}{\varphi_0 H_0^2}}$

According to BD theory, φ is the reciprocal of the effective Newtonian gravitational constant G. Therefore, the value of the parameter $\varphi_0 = \frac{1}{G_0} = 1.498 \times 10^{10}$ $N^{-1}m^{-2}Kg^2$. The values of other constants are, $H_0 = 2.33 \times 10^{-18} sec^{-1}$, $\rho_0 = 2.83 \times 10^{-27} Kg/m^3$ and $t_0 = 4.41 \times 10^{17} sec$, where H_0 and ρ_0 are the values of the Hubble parameter and the density of matter (both dark and baryonic) at the present epoch (t_0) [25]. Using the values of these constants in equation (13), the range of values of n is obtained as $-2.185 < n < -1.814$. For the present study, we have used two values of n ($n = -1.82$ and -1.88) within this range, chosen in a manner such that the time evolutions of the scale factor (a) and the deceleration parameter (q) are consistent with the observations. The plot of q versus $t - t_0$ has been shown in Figure 2 for two values of n. It is evident from this plot that the value of q is positive initially, becoming zero at a certain value of $t - t_o$ and it gradually attains a negative value at a value of t for which $t < t_o$. It is also observed in the same figure that the value of the deceleration parameter (q) remains negative even at the current epoch and thereafter. One may choose different values of n, within the obtained range, to just fine tune the value of $t - t_o$ at which the signature flip of q occurs, without affecting the nature of the plot.

4. FUNCTIONAL FORM OF THE BRANS-DICKE PARAMETER ω(φ)

Combining equations (3), (4), (5) and the expression of $\rho = \rho_0/a^3$, one obtains the following expression for the dimensionless parameter ω(φ) of BD theory.

$$\omega(\varphi) = \frac{2}{n^2}\left\{3(1+n) - \frac{\rho_0}{\varphi_0} a^{-(3+n)} \left(\frac{\dot{a}}{a}\right)^{-2}\right\} \tag{14}$$

Using equation (10) in (14) we get,

$$\omega(\varphi) = \frac{1}{n^2}[\omega_1 - \omega_2] \tag{15}$$

where, $\omega_1 = 6(1+n)$

and, $\omega_2 = \frac{2\rho_0(2+n)(3+n)(\varphi/\varphi_0)^{1+3/n}}{{H_0}^2\varphi_0(2+n)(3+n) + 2\rho_0\{-1+(\varphi/\varphi_0)^{1+3/n}\}}$

Equation (15) shows that for $n = -1.85$, we have $\omega \approx -1.49$ for an extremely large value of φ and, for an extremely small value of φ, $\omega \approx -1.54$. Since n is negative, φ must be decreasing with time, in accordance with equation (5). The above analysis therefore shows that ω becomes more and more negative with time.

5. TIME EVOLUTION OF THE GRAVITATIONAL CONSTANT (G)

Since φ is found to be decreasing with time, the gravitational constant $G(\equiv 1/\varphi)$ must be increasing with time. This result is consistent with other recent studies [26-28]. Combining equations (5) and (10), we get the following expression for G.

$$G = \frac{1}{\varphi} = \frac{1}{\varphi_0} a^{-n} = \frac{1}{\varphi_0}[a_1 + a_2]^{\frac{-n}{3+n}} \tag{16}$$

where, $a_1 = 1 + H_0(3+n)(t - t_0)$

and, $a_2 = \frac{\rho_0(3+n)(t-t_0)^2}{2(2+n)\varphi_0}$

6. CONCLUSIONS

The present article explains the accelerated expansion of the universe in the framework of generalized Brans-Dicke theory. It clearly demonstrates that a time-dependent ω can indeed generate a signature flip in the deceleration parameter (q) in a matter dominated universe, unlike many other models based on Brans-Dicke theory. The plot, that shows the time dependence of q, clearly indicates a smooth change of the deceleration parameter from a positive to a negative value, in the recent past. Therefore, it is proved that the generalized BD theory is capable of explaining the observed cosmic acceleration, without taking into consideration any component that represents dark energy. This theory has an advantage due to the fact that the scalar field is already there as an integral part of the theory. Here, the dimensionless coupling parameter (ω) is found to have a negative value. Its functional dependence upon the scalar field (φ) has played an important role in making the effective pressure negative.

However, this theoretical study has its own shortcomings. Although one gets the correct nature of time evolution of the cosmological parameters, this model has failed to generate the correct value of the deceleration parameter for $t = t_0$. For better results, one may try different empirical expressions for φ, instead of the one represented by equation (5). Apart from that, the BD parameter (ω) has a low negative value and this is in contradiction to the local astronomical requirement of a high value of ω. This article, however, provides a direction in which one may carry out theoretical studies so that a suitable form of $\omega(\varphi)$ can be obtained, which would explain the cosmic acceleration properly and, at the same time, generate a value of ω that is consistent with the local astronomical observations.

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REFERENCES

[1]. Perlmutter, S. et al., 1997. Measurements of the Cosmological Parameters Omega and Lambda from the First 7 Supernovae at z >= 0.35. Astrophys.J., 483, 565-586.

[2]. Reiss, A.G. et al., 2004. Type Ia Supernova Discoveries at z > 1 from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution. Astrophys.J., 607, 665-687.

[3]. Sahni, V., Starobinsky, A.A., 2000. The Case for a Positive Cosmological Lambda-term. Int.J.Mod.Phys.D, 9(4), 373-444.

[4]. Sahni, V., 2002. The Cosmological Constant Problem and Quintessence. Class.Quant.Grav., 19, 3435-3448.

[5]. Vishwakarma, R.G., 2002. A Machian Model of Dark Energy. Class.Quantum.Grav., 19, 4747-4752.

[6]. Sil, A., Som, S., 2008. A Better Way to Reconstruct Dark Energy Models. Astrophys.Space.Sci., 318, 109-115.

[7]. Sen, A.A., Sethi, S., 2002. Quintessence Model With Double Exponential Potential. Phys.Lett.B, 532,159-165.

[8]. Banerjee, N., Das, S., 2005. Acceleration of the universe with a simple trigonometric potential. Gen.Relativ.Gravit., 37, 1695-1703.

[9]. Padmanabhan, T., 2002.Accelerated expansion of the Universe driven by tachyonic matter. Phys.Rev.D, 66, 021301-021304.

[10]. Chiba, T., Okabe, T., Yamaguchi, M., 2000. Kinetically driven Quintessence. Phys. Rev.D, 62, 023511-023528.

[11]. Gao, C., Chen, X., Shen, Y.G., 2009. Holographic Dark Energy Model from Ricci scalar curvature. Phys.Rev.D, 79, 043511-043517.

[12]. Zimbdahl, W., Pavon, D., 2004. Statefinder parameters for interacting dark energy. Gen.Rel.Grav., 36, 1483-1491.

[13]. Das, S., Corasaniti, P.S., Khoury, J., 2006. Super-acceleration as the signature of a dark sector interaction. Phys.Rev.D, 73, 083509-083517.

[14]. Banerjee, N., Das, S., Ganguly, K., 2010. Chameleon field and the late time acceleration of the universe. Pramana-Journal of Physics, 74, L481-L489.

[15]. Bertolami, O., Martins, P.J., 2000. Nonminimal coupling and quintessence. Phys.Rev.D, 61, 064007-064023.

[16]. Banerjee, N., Pavon, D., 2001. A Quintessence Scalar Field in Brans-Dicke Theory. Class.Quant.Grav., 18, 593-601.

[17]. Banerjee, N., Das, S., 2006. A late time acceleration of the universe with two scalar fields: many possibilities. Mod.Phys.Lett.A, 21 (35), 2663-2670.

[18]. Das, S., Banerjee, N., 2006. An interacting scalar field and the recent cosmic acceleration. Gen.Relativ.Gravit, 38, 785-794.

[19]. Clifton, T., Barrow, J.D., 2006. Decaying Gravity.Phys.Rev.D, 73, 104022-104034.

[20]. Moraes, P.H.R.S., Ribeiro, G., Correa, R.A.C., 2016. A transition from a decelerated to an accelerated phase of the universe expansion from the simplest non-trivial polynomial function of *T* in the *f(R, T)* formalism. Astrophys. Space Sci., 361, 227-231.

[21]. Roy, S., 2015. A Study of the Time Dependence of Deceleration Parameter and Gravitational Constant on the basis of Brans-Dicke Theory. International Journal of Science, Environment and Technology, 4(6), 1503-1512.

[22]. Bergmann, P.G., 1968. Comments on the Scalar-Tensor Theory. Int.J.Theor.Phys., 1, 25-36.

[23]. Wagoner, R.V., 1970. Scalar Tensor Theory and Gravitational Waves. Phys.Rev.D, 1, 3209-3216.

[24]. Brans, C., Dicke, R.H., 1961. Mach's Principle and a Relativistic Theory of Gravitation. Phys.Rev., 124, 925-935.

[25]. Padmanabhan, T., 2003. Cosmological Constant - the Weight of the Vacuum. Phys.Rept., 380, 235-320.

[26]. Roy, S., 2019. Time evolution of the matter content of the expanding universe in the framework of Brans-Dicke gravity. Research in Astronomy and Astrophysics, 19 (4), 61-74.

[27]. Roy, S., 2018. Time Dependence of the Proportions of Matter and Dark Energy of the Universe in the Framework of Brans-Dicke Theory. Int. J. Adv. Sci. Eng., 5 (2), 886-895.

[28]. Banerjee, N., Ganguly, K., 2009. Generalised scalar-tensor theory and the cosmic acceleration. Int.J.Mod.Phys.D, 18, 445-451.

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